Vertical Contracts in a Supply Chain and the Bullwhip Effect

Model, Extensions and Possible Applications to Trade

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Based on:

Qu, Zhan, Horst Raff and Nicolas Schmitt, Incentives Through Inventory Control in Supply Chains, *International Journal of Industrial Organization 59 (2018),* 486-513.

Qu, Zhan and Horst Raff, Vertical Contracts in a Supply Chain and the Bullwhip Effect, *Management Science*, forthcoming. (https://doi.org/10.1287/mnsc.2020.3630)

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Introduction

Bullwhip effect: Amplification of demand shocks as they pass upstream through the supply chain.

Example:

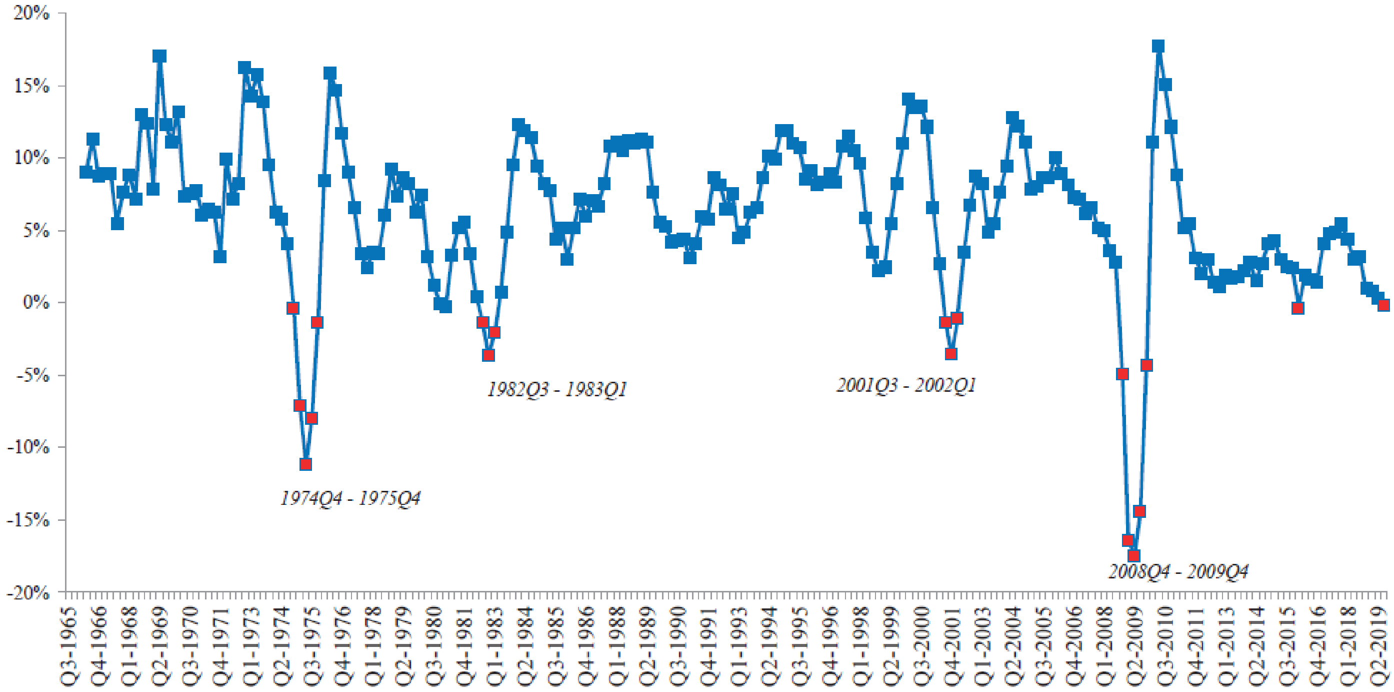
- Suppose a retailer sees a surge in demand for toilet paper and his inventory is running low.
- If he believes the demand will continue to be strong, he will have to place an order not only to replenish his inventory but also to have enough toilet paper on hand to satisfy high future demand.
- He orders more from his supplier than he currently sells.
- Supplier sees a greater demand shock than the retailer.
- And so on as we move up the chain.

- **Hence: variance of upstream production is greater than the variance of downstream sales**

Introduction

Great Trade Collapse

Figure 3 Quarter-on-quarter growth, world imports volume, 1965 to 2019 Q3



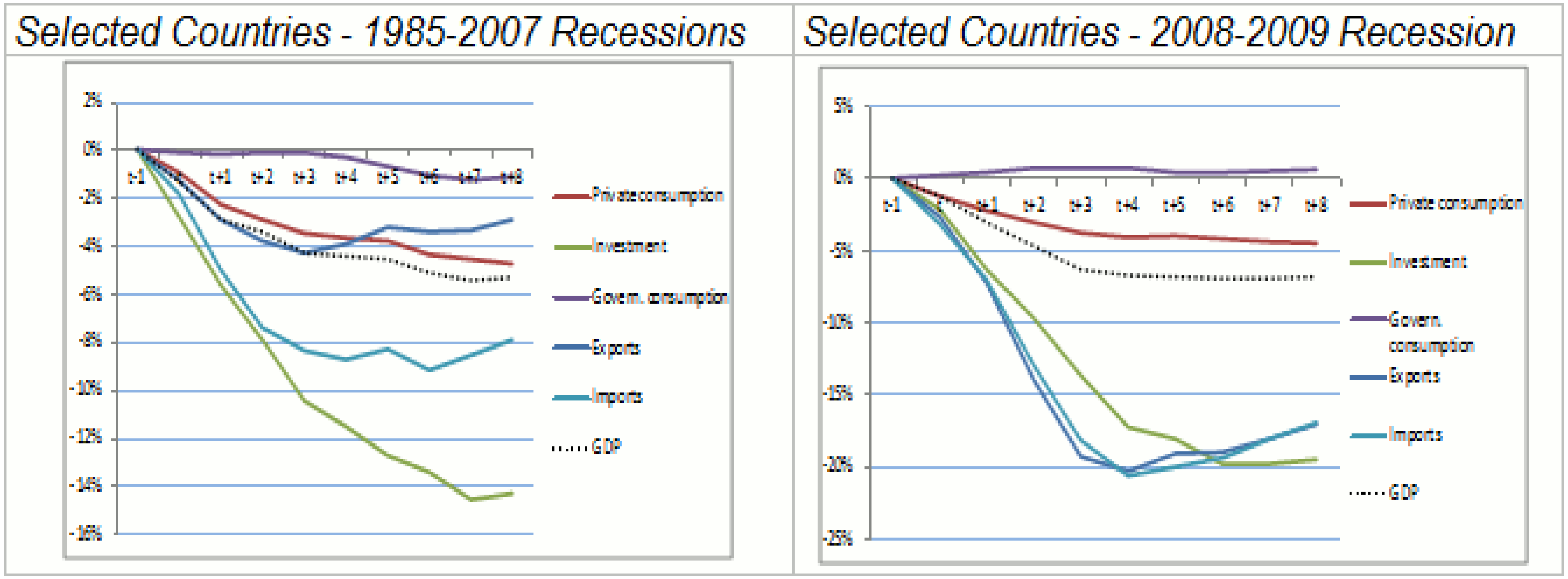
Source: Authors' elaboration on WTO online data; www.WTO.org.

Source: Baldwin and Weder di Mauro, 2020

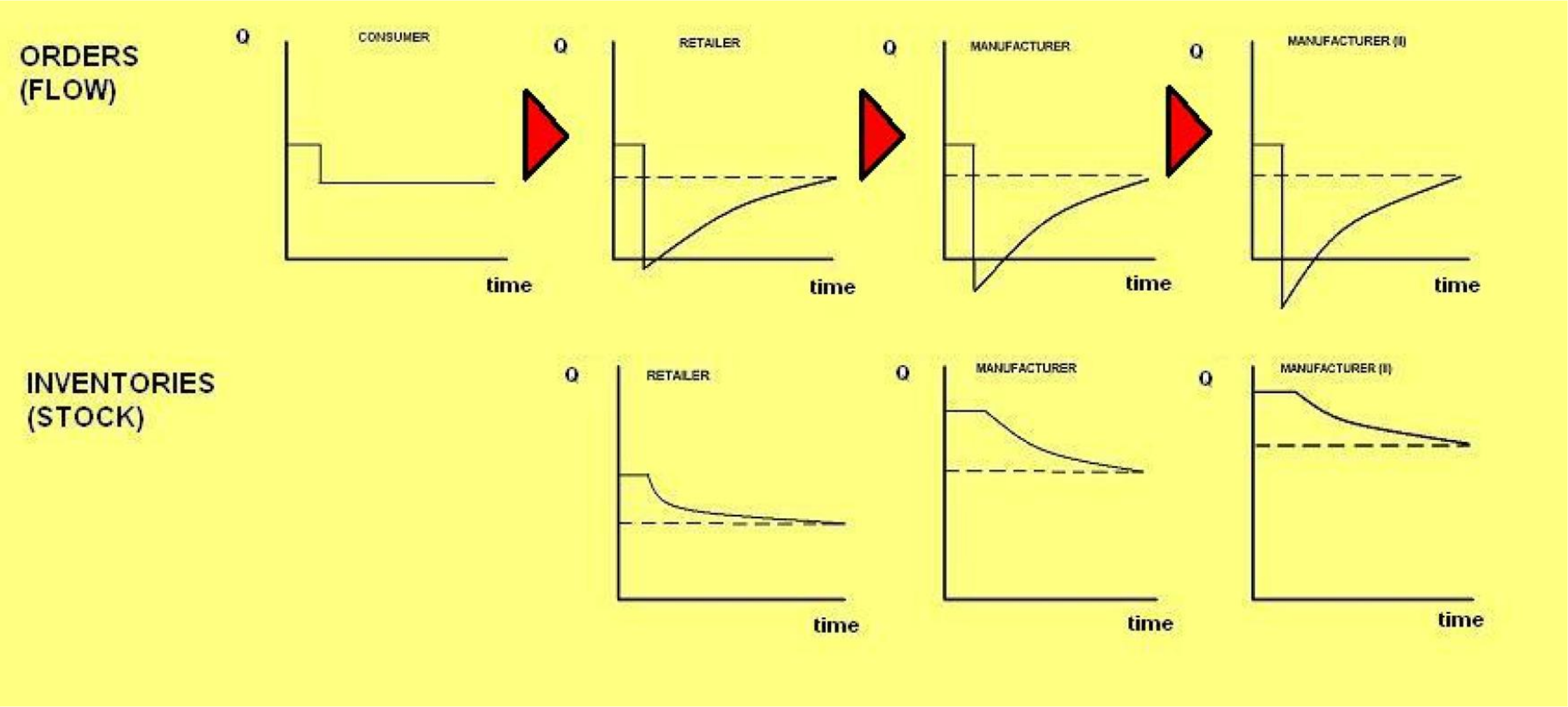
Introduction

Great Trade Collapse

- Alessandria et al. (2010, 2011), Zavacka (2012), Altomonte et al. (2013), Yilmazkuday (2019), Novy and Taylor (2019), Baldwin and Weder di Mauro (2020) argue that it represents at least in part a bullwhip effect:
- Trade declined more strongly and subsequently recovered much faster than demand (GDP) or private consumption as firms adjust their inventories



Introduction



Source: Altomonte et al., 2013

Introduction

Empirical Evidence for the Bullwhip Effect

- Macroeconomists find that production is more volatile than demand.
- Rather than smoothing production, inventory investment is positively correlated with sales: production counter-smoothing.
- Blinder and Maccini (1991) argue that 87% of the drop in GNP during the average postwar recession in the United States was accounted for by the fall in inventory investment.
- Industry level evidence for the US comes from Blinder and Maccini (1991) for various industries, Blanchard (1983) and Kahn (1992) for the automobile industry, Blinder (1981) for retailing, West (1986) for aggregate manufacturing, and more recently from Wen (2005a) and Cachon et al. (2007).
- Firm-level evidence from Bray and Mendelson (2012).
- Numerous case studies going back at least to Forrester (1961), and Holt et al. (1968).

Introduction

Modelling the Bullwhip Effect

- Kahn (AER 1986) for a monopolist, Lee et al. (Mgmt Science 1997) for a supply chain show that the bullwhip effect may arise if demand shocks are persistent.
- Intuitive explanation:
  - If demand today turns out to be high, firms respond by raising sales and by placing orders over and above what would be needed to replenish inventory, because they take the positive demand shock as a signal that demand will also be high tomorrow.
  - If demand today turns out to be low, firms reduce sales and run down inventory in anticipation of low demand tomorrow.
  - Persistent shocks imply that inventory investment is positively correlated with sales so that the variance of orders exceeds the variance of sales.

Introduction

Simple supply chain model:

- Upstream firm (manufacturer) sells goods to a downstream firm (retailer) who then sells to final consumers.
- Final demand is random, demand shocks may persist.
- Due to a production/transportation lead time, goods have to be ordered and produced before demand is known.
- Goods not sold this period can be stored and sold next period.

Vertical contracts:

- Lee et al. (1997) and essentially all supply chain models of the bullwhip effect (i) take producer and retail prices as fixed; and (ii) do not consider vertical contracts.
- But supply chains are plagued by vertical price and inventory distortions.
- Our research:
  - producer and retail prices are endogenous
  - consider the role of vertical contracts.

Introduction

Vertical contracts to deal with price and inventory distortions:

- Huge literature in economics and operations management dealing with vertical price and inventory distortions.
- But, to our knowledge, nobody has examined what these distortions and the contracts to solve them mean for the bullwhip effect.
- Vertical price distortions: double marginalization can be solved through a two-part tariff.
- Vertical inventory distortions:
  - How do you get retailers to hold the efficient amount of inventory?
  - Anand et al. (Mgmt Science 2008) show that the retailer may hold too much inventory.
  - Krishnan and Winter (AER 2007, Mgmt Science 2010) show that retailer may hold too little inventory.
- Problem vertical price and inventory distortions may interact, and the price system generically fails to solve them.

Introduction

Vertical contracts and the bullwhip effect:

Research Questions

1. What happens to the bullwhip effect if prices are endogenous?
2. How does the likelihood and amplitude of the bullwhip effect depend on the vertical contracts in a supply chain?
3. Suppose vertical integration solves vertical price and inventory distortions. Does this also imply a smaller bullwhip effect than in a decentralized supply chain with distortions?
4. Which contract form (vertical integration versus decentralized decision making) is better at hedging demand uncertainty and dampening the bullwhip effect?
5. Is there a tradeoff between efficiency of the supply chain and its resilience to demand uncertainty?
6. How do measures to dampen the bullwhip effect affect supply chain profit and the distribution of profits between manufacturers and retailers?
7. Should the bullwhip effect be eliminated or is there an optimal degree of the bullwhip effect?
8. ……

Vertical Contracts and the Bullwhip Effect

Model:

- Demand in period $t = 1,2$ is given by $p(s_t, \varepsilon_t) = 1 - s_t + \varepsilon_t$
  - $\varepsilon_1 = v_1$ and $\varepsilon_2 = v_2 + \rho\varepsilon_1$,
  - v_t uniformly distributed on $[-\Delta, \Delta]$,
  - $\rho \in [0,1]$ measures persistence.

- Marginal production cost: c
- Marginal cost of retailing = 0, no discounting, no cost of holding inventory across periods.
- Compare bullwhip effect in a vertically integrated supply chain and a decentralized supply chain with linear wholesale prices.
- Vertical integration solves vertical price and inventory distortions, but linear wholesale price in a decentralized supply chain does not.

Vertical Contracts and the Bullwhip Effect

Sequence of decisions in a vertically integrated supply chain:

- Period 0:
  - Manufacturer chooses how much to produce q_0
- Period 1:
  - q_0 becomes available, manufacturer observes ε_1
  - manufacturer chooses sales $s_1 \leq q_0$
  - Manufacturer chooses how much to produce q_1
- Period 2:
  - q_1 becomes available, manufacturer observes ε_2
  - manufacturer chooses sales $s_2 \leq q_1 + (q_0 - s_1)$

Vertical Contracts and the Bullwhip Effect

Definitions:

- Bullwhip effect: $Var[q_1(\varepsilon_1)] > Var[s_1(\varepsilon_1)]$
- Production smoothing: $Var[s_1(\varepsilon_1)] > Var[q_1(\varepsilon_1)]$

Lemma 1. *Let* $t_1(\varepsilon_1)$ *and* $t_2(\varepsilon_1)$ *be functions of* ε_1. *If* $t_1'(\varepsilon_1) \geq t_2'(\varepsilon_1) \geq 0$ *for any* ε_1 *and the support of* $t_1'(\varepsilon_1) - t_2'(\varepsilon_1)$ *has a nonzero measure, then* $\mathrm{Var}[t_1(\varepsilon_1)] > \mathrm{Var}[t_2(\varepsilon_1)]$

According to Lemma 1, a sufficient condition for a bullwhip effect is:

$$\frac{\partial q_1}{\partial \varepsilon_1} > \frac{\partial s_1}{\partial \varepsilon_1} > 0$$

Vertical Contracts and the Bullwhip Effect

Simplifying Assumption: variance of demand is sufficiently small so that in equilibrium there are no stockouts in period 1 and no unsold inventory at the end of period 2.

Assumption 1.

$$\Delta < \min\left\{c, \frac{1-c}{3(2-\rho)}\right\}.$$

Vertically integrated supply chain:

- Since $s_2 = q_1 + (q_0 - s_1)$ and q_0 is chosen before ε_1 is known:

$$\frac{\partial s_2}{\partial \varepsilon_1} = \frac{\partial q_1}{\partial \varepsilon_1} - \frac{\partial s_1}{\partial \varepsilon_1}.$$

- Hence a bullwhip effect exists if:

$$\frac{\partial s_2}{\partial \varepsilon_1} > 0 \text{ and } \frac{\partial s_1}{\partial \varepsilon_1} > 0$$

Vertical Contracts and the Bullwhip Effect

- In period 2 the manufacturer maximizes expected profit:

$$\max_{s_2} \{(1 - s_2 + \rho\varepsilon_1)\, s_2 - c\,[s_2 - (q_0 - s_1)]\}$$

- Hence

$$s_2 = \frac{1-c+\rho\varepsilon_1}{2} \qquad \frac{\partial s_2}{\partial \varepsilon_1} = \frac{\rho}{2}$$

- In period 1 we obtain:

$$s_1 = \frac{1-c+\varepsilon_1}{2} \qquad \frac{\partial s_1}{\partial \varepsilon_1} = \frac{1}{2} > 0$$

Proposition 1. *If Assumption 1 holds, then a vertically integrated supply chain experiences a bullwhip effect for any* $\rho > 0$.

Vertical Contracts and the Bullwhip Effect

Sequential game of complete information in a decentralized supply chain:

- Period 0:
  - Manufacturer chooses wholesale price P_{r0}
  - Retailer orders q_{r0}

- Period 1:
  - q_{r0} becomes available, manufacturer and retailer observe ε_1
  - Retailer chooses sales $s_{r1} \leq q_{r0}$ and manufacturer chooses P_{r1} simultaneously
  - Retailer orders q_{r1}

- Period 2:
  - q_{r1} becomes available, manufacturer and retailer observe ε_2
  - retailer chooses sales $s_{r2} \leq q_{r1} + (q_{r0} - s_{r1})$

Vertical Contracts and the Bullwhip Effect

Decentralized supply chain:

- The retailer's optimal sales in period 2 are given by:

$$s_{r2} = \frac{1 - P_{r1} + \rho\varepsilon_1}{2}$$

- Hence: $$\frac{\partial s_{r2}}{\partial \varepsilon_1} = \frac{\rho}{2} - \frac{1}{2}\frac{\partial P_{r1}}{\partial \varepsilon_1}$$

- The price effect is positive: $$\frac{\partial P_{r1}}{\partial \varepsilon_1} > 0.$$

Proposition 2. *If Assumption 1 holds, then a decentralized supply chain experiences production smoothing for any $\rho < 0.5$ and a bullwhip effect for any $\rho > 0.5$. The bullwhip effect is increasing in ρ.*

Proposition 3. *If Assumption 1 holds, then the variances of both production and sales are smaller in a decentralized supply chain than in a vertically integrated supply chain.*

Vertical Contracts and the Bullwhip Effect

These results hold if:

- there are stockouts in period 1 and/or unsold inventory at the end of period 2,
- there are more than two demand periods,
- production costs are convex instead of linear,
- demand is non-linear,
- demand shock is non-uniform,
- retailer chooses sales before the manufacturer announce wholesale price in period 1,
- there is discounting and/or inventory holding cost,….

They also hold if the manufacturer uses a two-part tariff and thus non-linear wholesale price (i.e. quantity discounts)

- but the mechanism that dampens the bullwhip effect is entirely different.

Vertical Contracts and the Bullwhip Effect

Let's take stock:

- The volatility of production and sales and the size of the bullwhip effect are smaller in a decentralized than in a vertically integrated supply chain.
- This rationalizes the finding of Altomonte et al. (2013) that during the Great Trade Collapse the bullwhip effect was stronger for intra-firm trade within French multinationals than for arm's-length trade.
- Demand shocks probably have different effects on trade depending on the form of trade: intra-firm trade, processing trade, arm's-length trade.

Vertical Contracts and the Bullwhip Effect

What can we say about overall supply chain profit and the distribution of profits between manufacturer and retailer?

Is there a tradeoff between efficiency and resilience?

- A vertically integrated supply chain always makes greater expected profit than a decentralized supply chain, if it can solve all externalities.
- How does dampening the bullwhip via the price effect impact on expected profits in a decentralized supply chain?
- To answer this question we need a counterfactual, namely a decentralized supply chain without price effect, as would happen if the manufacturer did not observe ε_1.

Vertical Contracts and the Bullwhip Effect

Proposition 4. *(i) Consider a decentralized supply chain. The price effect increases the manufacturer's expected profit and decreases the retailer's expected profit as well as the supply chain's aggregate expected profit. These changes in profit are greater the greater the variance and the persistence of demand shocks are. (ii) Double marginalization implies that a decentralized supply chain's expected profit even without price effect is lower than that of a vertically integrated supply chain.*

- Dampening the bullwhip effect benefits the manufacturer but hurts the retailer and comes at the expense of aggregate supply chain profit.
- From the point of view of aggregate supply chain profit the bullwhip effect should not be dampened,
- at least in the absence of explicit costs associated with the bullwhip effect.

Vertical Contracts and the Bullwhip Effect

Problem:

- We have assumed that the manufacturer in a decentralized supply chain has only one instrument, namely the wholesale price, to earn profits and to dampen the bullwhip effect.
- Clearly this is not enough to eliminate double marginalization and solve vertical inventory distortions.
- Maybe it is not surprising that dampening the bullwhip effect is not profitable for the supply chain, as the manufacturer uses the price effect mostly to better price discriminate.

- Suppose we let the manufacturer use a two-part tariff (T_t, w_t)

Vertical Contracts and the Bullwhip Effect

Sequential game of complete information in a decentralized supply chain:

- Period 0:
  - Manufacturer chooses (T_0, w_0)
  - Retailer orders q_0

- Period 1:
  - q_{r0} becomes available, manufacturer and retailer observe ε_1
  - Retailer chooses sales $s_1 \leq q_0$ and manufacturer chooses(T_1, w_1) simultaneously
  - Retailer orders q_1

- Period 2:
  - q_1 becomes available, manufacturer and retailer observe ε_2
  - retailer chooses sales $s_2 \leq q_1 + (q_0 - s_1)$

Vertical Contracts and the Bullwhip Effect

- We know from Anand et al. (Mgmt Science 2008), that the retailer will want to place a large order q_0 and keep s_1 small so that he can carry a large inventory $(q_0 - s_1)$ into period 2.
- $(q_0 - s_1)$ **strategic inventory**
- The reason is that the retailer can guarantee himself a revenue $\hat{R}$ in period 2 that the manufacturer cannot capture through T_1:

$$\hat{R} \equiv [1 - (q_0 - s_1) + \rho\varepsilon_1](q_0 - s_1)$$

- There is by now a sizable literature on strategic inventory and even experimental evidence that it is important in practice.
- We want to show that there is a natural connection between strategic inventory and the bullwhip effect.

Vertical Contracts and the Bullwhip Effect

Why does strategic inventory dampen the bullwhip effect?

- The bigger is q_0 the more likely it is that the retailer does not order anything in period 1, ie. $q_1 = 0$.
- But since q_0 cannot depend on ε_1, but sales do depend on ε_1, we necessarily obtain production smoothing as long as q_1 and ρ are sufficiently small.

Two obstacles to show this formally:

1. The bullwhip effect could also be dampened by the price effect.
2. Due to demand uncertainty the retailer may not only carry strategic inventory but also safety inventory to prevent stocking out. In fact we have already shown that a vertically integrated supply chain will typically carry safety inventory, but obviously no strategic inventory.

Vertical Contracts and the Bullwhip Effect

How to deal with these obstacles

1. We can prove that for $q_1 > 0$ the manufacturer will choose $q_1 = c$. Hence no price effect.
2. Assume that Δ is so small that the retailer always stocks out in period 2. Hence he does not need to carry any safety inventory.

We find that in equilibrium in period 1:

$$s_1 = \frac{1 - w_0}{2} + \frac{(1-\rho)\,\varepsilon_1}{4} \quad \text{and} \quad q_1 = \frac{2\,(w_0 - c) + (1+\rho)\,\varepsilon_1}{4}$$

If $w_0 \geq \hat{w}_0$ and thus big enough to guarantee $q_1 > 0$, we obtain a bullwhip effect for any $\rho > 0$:

$$\frac{\partial q_1}{\partial \varepsilon_1} = \frac{1+\rho}{4} > \frac{\partial s_1}{\partial \varepsilon_1} = \frac{1-\rho}{4} \geq 0$$

Vertical Contracts and the Bullwhip Effect

- Note that $$\hat{w}_0 = c + \frac{1}{2}(1+\rho)\Delta$$
- If $w_0 \le \breve{w}_0$ then $q_1 = 0$, we obtain a production smoothing for any $\rho > 0$:
$$\breve{w}_0 - c = -\frac{1}{2}(1+\rho)\Delta$$
- Hence it is not optimal to eliminate the bullwhip effect.
- The optimal bullwhip effect is determined by the w_0 that maximizes total expected profit:

$$\max_{w_0} (w_0 - c)(1 - w_0) + E\left[\frac{(1-w_0)^2}{2} + \frac{(1-\rho)^2 \varepsilon_1^2}{8}\right]$$

$$+ \int_{\hat{\varepsilon}_1}^{\Delta} \left[\frac{(1-c+\rho\varepsilon_1)^2}{4} + c(q_0 - s_1) - \hat{R}\right] \frac{1}{2\Delta} d\varepsilon_1,$$

Vertical Contracts and the Bullwhip Effect

- The optimal w_0 is greater than c but lower than $\widehat{w}_0$, namely

$$w_0^* = c + \left(\frac{3}{2} - \sqrt{2}\right)(1+\rho)\,\Delta$$

- The optimal bullwhip effect reflects a tradeoff between using w_0
  - to eliminate the vertical price distortion: ideally $w_0 = c$
  - to eliminate the vertical inventory distortion: $w_0 \geq \widehat{w}_0$

- Finally we can prove that these results also hold if we allow the retailer to carry safety inventory.

Vertical Contracts and the Bullwhip Effect

Conclusions

- The likelihood and amplitude of the bullwhip effect depends on the vertical contracts in the supply chain.
- Empirically testable prediction: vertically integrated supply chains (MNEs, contract processing?) are more likely to exhibit a bullwhip effect than decentralized supply chains (arm's-length trade?). Their bullwhip is also larger.
- Prescriptive implication: manufacturers benefit from dampening the bullwhip effect by adjusting wholesale prices (flexible price contract). But this may hurt retailers and reduce overall supply chain profit.
- If we allow for two-part tariffs (quantity discounts), there is an optimal bullwhip effect that maximizes supply chain profit.
- The optimal bullwhip reflects a tradeoff between using w_0 to eliminate the vertical price distortion and using it to eliminate the vertical inventory distortion.