



## KCG Working Paper

# Incentives through Inventory Control in Supply Chains<sup>1</sup>

Zhan Qu, Horst Raff and Nicolas Schmitt

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**Abstract:** The paper shows that taking inventory control out of the hands of competitive or exclusive retailers and assigning it to a manufacturer increases the value of a supply chain especially for goods whose demand is highly volatile. This is because doing so solves incentive distortions that arise when retailers have to allocate inventory across sales periods, and thus allows for better intertemporal price discrimination. Assigning inventory control to a manufacturer is also shown to have effects on total inventory and social welfare.

**Keywords:** inventory, supply chain, demand uncertainty, storable good, price discrimination

**JEL Classification:** L11, L12, L81

Zhan Qu  
Department of Economics  
Technical University Dresden  
Helmholtzstraße 10  
01069 Dresden  
Germany  
zhan.qu@tu-dresden.de

Horst Raff  
Kiel Centre for Globalization  
University of Kiel  
Wilhelm-Seelig Platz 1  
24118 Kiel  
Germany  
raff@econ-theory.uni-kiel.de

Nicolas Schmitt  
Department of Economics  
Simon Fraser University  
CESifo  
8888 University Drive  
Burnaby BC V5A 1S6  
Canada  
Schmitt@sfu.ca

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# 1 Introduction

The optimal control of inventory is one of the greatest challenges faced by firms in a supply chain. Consider, for instance, a supply chain, in which a manufacturer distributes goods through retailers and the need to hold inventory arises, because goods have to be produced before the state of demand is known and sales to consumers can take place. A key problem in this setting, as explained by Krishnan and Winter (2007), is that the manufacturer's and retailers' incentives to hold inventory are generally not aligned. Hence the challenge is how to solve such incentive problems so that the supply chain's value can be maximized.

In the current paper, we examine the incentives that arise when inventory has to be allocated intertemporally, because there is more than one sales period, and inventory control rests with either competitive retailers, an exclusive retailer, or the manufacturer. Specifically we show that there are two incentive distortions when competitive retailers control inventory, both associated with multiple sales periods. We also show that delegating inventory control to an exclusive retailer instead of competitive retailers resolves these distortions, provided that the manufacturer uses two-part tariffs, but replaces them with two new ones, also associated with multiple sales periods but specific to the exclusive retailer. Thus, while incentives are better aligned between a manufacturer and an exclusive retailer than between a manufacturer and competitive retailers, it is only when the manufacturer controls inventory itself that these distortions disappear. Our analysis thus indicates that, based on incentive considerations alone, there is a benefit to having the manufacturer control inventory rather than assigning inventory control to retailers.

The question of who in a supply chain should control inventory is important in practice. Consider, for example, O'Neill Inc., a US manufacturer of apparel and accessories for water sports. Production takes place in Asia to take advantage of low costs. But due to the long lead time (3 months), production has to occur well before the demand is known. O'Neill allows for two types of orders from US retailers (Cachon, 2004): one placed well before the selling season (with delivery guarantee), which means that retailers take possession of the goods, control inventories and manage stocks over time in the face of fluctuating demand. The other type of order can be made on short notice during the selling season and is honored provided inventory is available in O'Neill's distribution center in San Diego. In this case, it is the manufacturer itself who controls inventory and who has to make sure that enough inventory is on hand to meet demand by retailers. Another example is Trek Inc., a US bicycle manufacturer, who lets its exclusive retailers control the inventory of some of its most popular products but entirely controls the inventory of its high-end bicycles, for which demand is particularly uncertain (Cachon, 2004).

More generally, we observe a broader trend to take inventory control out of the hands of retailers and pass it upstream to manufacturers or wholesalers, as evidenced by the widespread adoption of business practices, such as ‘drop shipping’, ‘inventory consignment’, and ‘vendor-managed inventory’. Drop-shipping is an arrangement used especially by internet retailers, where these retailers forward buyers’ orders to a manufacturer or wholesaler who then ships the product from its own inventory directly to the buyer. Inventory consignment allows an upstream firm to own inventories held by downstream firms, while vendor-managed inventory (VMI) allows a manufacturer or wholesaler to manage these inventories.<sup>1</sup> Already in 2002, 30% of internet retailers used drop shipping as their main means of order fulfillment compared with 5% of regular multi-channel retailers (Randall et al., 2002).

What these examples show is that, thanks in part to advances in information and communication technologies, including in electronic sale and inventory tracking, it is now possible for manufacturers to control inventories in a more cost-effective way than in earlier times when they were forced to let retailers control inventories. Hence it is now more important than ever to understand how the incentives to manage inventory differ when either retailers or manufacturers are in control.

To examine these incentive issues we use a standard model of a supply chain, in which a manufacturer distributes goods through retailers, and goods have to be produced before demand is known; all sales are hence from inventory. The novelty is that we explicitly focus on the decision to allocate inventory intertemporally through a model with two sales periods, and that we compare the cases where inventory is controlled either by competitive retailers, an exclusive retailer, or the manufacturer itself. Examining both competitive retailers and an exclusive retailer, for which the manufacturer uses a two-part tariff, allows us not only to compare the two opposite extremes of retail market structures, namely perfect competition and monopoly, but also to focus on the cases in which there is, at least in principle, no double marginalization and thus no ex-ante vertical price distortion that could obscure the incentive effect of inventory control. To keep the focus plainly on incentives through inventory control, we also abstract from any cost advantages that may favor the centralization of inventory control, such as economies of scale, cost advantages through the provision of complementary services such as transportation, and economies of scope arising from pooling inventory control for different products.

We show that two incentive distortions arise when competitive retailers control inventory. The first has to do with the intertemporal allocation of inventory. Competitive retailers allocate inventory so that today’s retail

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<sup>1</sup>See Randall, Netessine and Rudi (2006) for a recent comparison of drop shipping with the case of retailer-controlled inventory, Govindam (2013) for a survey, and also Mateen and Chatterjee (2015).

price is equalized with tomorrow's expected retail price. When retail prices in the two periods are tied to each other in this way, they cannot adjust sufficiently to demand conditions in each period, which impedes intertemporal price discrimination.

The second problem associated with competitive retailers is that the excess inventory they carry into period two reduces the residual demand faced by the manufacturer and hence also the producer price in that period. In effect, the manufacturer competes with the excess inventory carried over by retailers from period one. To limit the impact of this inventory competition the manufacturer would have to keep shipments in period one small, but by doing so it runs the risk of losing sales due to stockouts. It is to avoid these stockouts that the manufacturer has to ship more than would be optimal without the distortion.

Next we show that, by assigning inventory control to an exclusive retailer and using a two-part tariff, the manufacturer can eliminate inventory competition, while at the same time avoiding stockouts. On this count then the exclusive retailer's incentives when it controls inventory are better aligned with the interests of the manufacturer than those of competitive retailers. However, having an exclusive retailer control inventory creates two distortions of its own so that intertemporal price discrimination is still suboptimal. One distortion comes from the fact that the exclusive retailer does not optimally allocate inventory across periods. In particular, we show that the retailer has an incentive to carry too much excess inventory into period two and therefore to order too little in that period. The manufacturer responds to this by raising the period-one producer price above marginal cost, which leads to a second distortion in the form of double marginalization. In other words, whether the retailers are competitive or exclusive and whether the manufacturer uses linear or two-part pricing, there are still vertical distortions when retailers control inventories. We show that these vertical distortions become worse the higher is the variance of demand. Thus, from the manufacturer's point of view, taking inventory control away from retailers is especially useful in markets where final demand is very volatile.

However, while letting the manufacturer control inventory raises the overall expected profit of the supply chain, the effect on expected consumer surplus and social welfare depends on retail market structure. Expected consumer surplus and social welfare fall when inventory control is passed from competitive retailers to the manufacturer. Hence, in the absence of major cost savings from shifting inventory control to the manufacturer, such a move should be viewed as being anticompetitive. The opposite effect on consumer surplus and social welfare is obtained when inventory control is transferred to the manufacturer from an exclusive retailer.

Incentive problems in decentralized supply chains have been analyzed by the literature on vertical control in industrial organization and by the man-

agement literature on supply chain coordination.<sup>2</sup> In particular, Krishnan and Winter (2007) and Deneckere et al. (1996) explain that the price system generically fails to align retailers' incentives with those of the manufacturer when inventory control is involved. Our model is closely related to the one by Deneckere et al. (1997), in which a manufacturer sells goods through competitive retailers who control inventory. Unlike in our paper, inventory in Deneckere et al. perishes after one period, which implies that retailers will sell their entire inventory, even if the retail price drops to zero. It is this risk of 'destructive competition' that explains why retailers may hold too little inventory, and it is to avoid this effect that the manufacturer may want to intervene, for instance, by imposing resale price maintenance or offering buy-back policies.

By contrast, we deliberately choose to eliminate destructive competition by allowing retailers to shift unsold inventory into the second period and by assuming that the demand shocks are sufficiently small so that the retail price never hits zero in period 2. This allows us to focus on the intertemporal incentive problems associated with retailer-controlled inventory. As already mentioned above, these incentive problems imply that retail prices do not adjust optimally to demand conditions, so that intertemporal price discrimination is impeded. Moreover, competitive retailers tend to order too much inventory from the manufacturer's point of view, which is in stark contrast to the findings by Deneckere et al. (1997).

Another departure from Deneckere et al. is that we consider how incentives would change if control over inventory were assigned to an exclusive retailer. In the Deneckere et al. model, having an exclusive retailer in control of inventory and imposing a two-part tariff would solve the problem of destructive competition and lead to an optimal inventory policy. In our model, by contrast, incentive problems persist even with an exclusive retailer, meaning that intertemporal price discrimination is still not optimal. Only when the manufacturer takes control of inventory can the intertemporal allocation distortions be eliminated in our model. However, this reduces social welfare with respect to the case where competitive retailers hold inventory. By contrast, eliminating incentive distortions in Deneckere et al. raises social welfare, as inventory rises.

Another related paper is by Krishnan and Winter (2010). In their model a manufacturer distributes goods through two retailers, and excess inventory may be carried over into the subsequent period. They show that, if inventory does not perish too quickly, retailers may hold too much inventory. This is

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<sup>2</sup>See Krishnan and Winter (2011) for a synthesis of these two strands of literature. There is also, of course, a long literature on optimal order policies and inventory levels going back to seminal contribution of Arrow, Harris and Marshak (1951). Clark and Scarf (1960) were the first to establish an optimal inventory policy in a multi-echelon supply chain. These papers, however, do not examine the incentive problems associated with inventory control.

similar to the results of our paper, but the intertemporal incentive distortions leading to this result are different from ours, in part because inventory in their model has itself a positive effect on consumer demand. Their paper also differs from ours in other respects. For instance, Krishnan and Winter do not examine how the incentives to hold inventory differ between competitive and exclusive retailers, and what the consequences of different inventory control arrangements are for social welfare.

Our paper is also related to the literature on intertemporal price discrimination (Varian, 1989; Bulow, 1982; Koh, 2006; Dudine et al., 2006). In this literature, consumers engage in strategic behavior whether it is to delay purchases (as with durable goods), or to stockpile goods when consumers anticipate an (exogenous) increase in prices (as with storable goods). Consumer's strategic behavior is thus at the heart of results whereby a firm competes with itself (as in the Coase conjecture in the durable good monopoly problem). By dealing with intertemporal incentives associated with inventory control at the firm level, we do not need consumer strategic behavior. As a result our inventory competition effect does not depend on such consumers.

The rest of the paper is organized as follows. In Sections 2 to 4 we present our model and derive the equilibrium if inventory is controlled by competitive retailers, an exclusive retailer and the manufacturer, respectively. Section 5 discusses some implications of the analysis, Section 6 contains conclusions, and the Appendix collects the proofs of our results.

## 2 The Model and Benchmark Case

Consider a supply chain consisting of a manufacturer selling goods through retailers. In the benchmark model we follow Deneckere et al. (1997) by assuming that retailers are perfectly competitive.<sup>3</sup> Retailers have to order and take possession of goods before demand is known. Once demand has been revealed, the retailers sell to consumers. Thus, the retailers hold inventories, that is to say the units received from the manufacturer and stored before they are sold.

We deviate from Deneckere et al. (1997) by assuming that there are two sales periods,  $t = 1, 2$ , which means that the product under consideration loses its value after two periods. Demand at time  $t = 1, 2$  is given by the linear inverse demand function:  $p_t = A - s_t + \varepsilon_t$ , where  $p_t$  is the retail price

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<sup>3</sup>The assumption of perfectly competitive retailers is not overly restrictive in the sense that one can view the competitive outcome as the limit of a sequential game among oligopolistic retailers as the number of retailers gets large. In stage one retailers order inventory, each taking the quantity of the other retailers as given (Cournot competition). In stage two, after observing the true realization of demand, retailers simultaneously announce retail prices. The outcome of the subgame perfect equilibrium of this game converges to the perfectly competitive outcome as the number of retailers goes to infinity. See Tirole (1988), ch. 5 and the references cited there for the relevant convergence results.



and  $s_t$  denotes final sales in period  $t$ . The random variables  $\varepsilon_t \in [-d, d]$  are intertemporally independent and uniformly distributed with density  $1/2d$ .

We keep the assumptions about the production and distribution technologies as simple as possible. The manufacturer incurs a constant unit cost of production  $c$ . The marginal cost of retailers is normalized to zero, as is the cost of holding inventory, and there is no discounting between periods. All market participants are risk neutral. We also assume  $A > c$ , and that the demand shock is not too big,  $d \leq \bar{d} = \min \left[ \frac{c}{\sqrt{2}}, \frac{A-c}{2} \right]$  so that equilibrium prices and sales in each period are always non-negative in all the environments considered in the analysis (see Appendix 6). Importantly, the latter assumption implies that, in equilibrium, all inventory remaining in period 2 is sold at a positive price and thus that destructive competition (Deneckere et al., 1996, 1997), another source of (static) incentive problems arising from retailer controlled inventory, is assumed away. Allowing for the possibility of destructive competition and/or more complicated cost structures would obscure the intertemporal incentive issues that we focus on.

The order of moves can now be summarized as follows. At the beginning of period 1, the manufacturer announces a producer price  $P_1$ , retailers order and take possession of  $q_1$  units of goods before demand in period 1 is known; then period-one demand is revealed and the retailers sell  $s_1 \leq q_1$  in period 1, holding unsold units in inventory for period 2. In period 2, the manufacturer sets producer price  $P_2$ , and retailers order quantity  $q_2$ , again before demand is known. Demand in period 2 is then revealed and retailers sell  $s_2 \leq q_2 + (q_1 - s_1)$ .

Now consider period 2. The retailers sell all of the products on hand because they have already paid for these goods and, for  $d \leq \bar{d}$ , the retail price is positive. Hence  $s_2 = q_2 + (q_1 - s_1)$ : sales in period 2 correspond to the sum of the excess inventory carried over from period 1 and the quantity ordered in period 2. Since retailers must order before the demand in that period is known, competitive retailers order goods in period 2 until the expected consumer price in period 2 equals the producer price  $P_2$  and thus until  $E(A - s_2 + \varepsilon_2) = A - q_2 - (q_1 - s_1) = P_2$ .

The manufacturer chooses  $P_2$ , respectively  $q_2$ , to maximize its period-2 expected profit  $E\{(P_2 - c)q_2\}$ . This expected profit is maximized for  $q_2 = (A - c - (q_1 - s_1))/2$  or  $P_2$  such that:

$$P_2 = E(p_2) = \begin{cases} \frac{A+c}{2} - \frac{(q_1-s_1)}{2} & \text{if } s_1 < q_1, \\ \frac{A+c}{2} & \text{otherwise.} \end{cases} \quad (1)$$

Thus, if all units ordered in period 1 are sold in that period ( $s_1 = q_1$ ), the manufacturer sets the producer price in period 2 equal to the static monopoly price. But if retailers carry excess inventory into period 2 ( $s_1 < q_1$ ), the manufacturer is forced to reduce its producer price below the static monopoly price. This price effect, which we refer to as *inventory competition*, can be

viewed as a vertical distortion coming directly from delegating inventory control to retailers. This distortion has implications for period 1. To see this, it suffices to recognize that orders in period 1,  $q_1$ , are critical to determine whether or not there is inventory competition in period 2, since the bigger is  $q_1$  the more likely it is that excess inventory will be passed on to period 2. Obviously the manufacturer wants to take into account how retailers make their choices in period 1 to determine the optimal producer price in that period,  $P_1$ .

Turning then to period 1, once the demand in that period has been revealed, retailers sell as long as the realized retail price,  $p_1(\varepsilon_1)$ , is greater than or equal to the expected retail price in period 2. Hence, sales in period 1,  $s_1$ , are determined by the condition:

$$p_1(\varepsilon_1) = E(p_2), \quad (2)$$

and thus by  $p_1(\varepsilon_1) = A - s_1 + \varepsilon_1 = \frac{A+c-q_1+s_1}{2} = E_2(p_2)$ . By linking prices in the two periods in this fashion competitive retailers impede intertemporal price discrimination. In fact, optimal price discrimination would have required a level of first-period sales that equates the marginal revenue in period 1 to the expected marginal revenue in period 2.<sup>4</sup> Given the distortion inherent in (2), it follows that

$$s_1 = \begin{cases} \frac{A-c+2\varepsilon_1}{3} + \frac{1}{3}q_1 < q_1 & \text{if } \varepsilon_1 < \hat{\varepsilon}_1, \\ q_1 & \text{otherwise,} \end{cases} \quad (3)$$

where

$$\hat{\varepsilon}_1 = q_1 - \frac{A-c}{2}. \quad (4)$$

Given  $q_1$ , (3) offers two possible outcomes for  $s_1$ . Either  $s_1 < q_1$  if there is a relatively low realization of demand leading to inventory competition in period 2, or else, if there is a relatively high realization of demand in period 1,  $s_1 = q_1$  and retailers stock out so that they do not carry any excess inventory into period 2.

If inventory competition is a problem for the manufacturer, having retailers stock out by selling in period 1 all the units ordered is also a problem since it implies that sales are being lost. It is obvious from (4) that the level of  $q_1$  determines the value of  $\hat{\varepsilon}_1$  and thus the range of demand realizations leading either to inventory competition in period 2 or stockout in period 1. The manufacturer has just one tool at its disposal,  $P_1$  or equivalently the choice of  $q_1$ , to balance these two effects. Formally, this trade-off is solved by choosing  $q_1$  to maximize the manufacturer's expected profit  $E\{(P_1 - c)q_1 + (P_2(q_1) - c)q_2(q_1)\}$ .

The following proposition shows how the manufacturer optimally deals with this trade-off, where the superscript '*cr*' denotes the expected equilibrium values in the case of competitive retailers:

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<sup>4</sup>We will come back to this distortion in the next sections.

**Proposition 1** *Suppose inventory is controlled by competitive retailers. The manufacturer, facing a trade off between lost sales in period 1 and inventory competition in period 2, chooses to sell in period 1 more than the one-period static monopoly output, namely  $q_1^{cr} = \frac{(A-c)}{2} + \frac{d}{5}$ , and to incur a 40% probability that retailers stock out.*

**Proof:** See Appendix 1.

To understand the intuition behind the manufacturer's choice of  $q_1$ , it is useful to rewrite the manufacturer's first-order condition derived in Appendix 1 (see eq. (A.5)) as follows:

$$2(A - c - 2q_1) + 7 \int_{\hat{\varepsilon}_1}^d \left( A - c - 2q_1 + \frac{8}{7}\varepsilon_1 \right) \frac{1}{2d} d\varepsilon_1 = 0. \quad (5)$$

The first term represents the maximization condition for expected profits assuming there is no possibility of a stockout. This term would be equal to zero at the static monopoly output of  $q_1 = (A - c)/2$ . That is, if there were no possibility of a stockout, the manufacturer would simply deliver the unconstrained optimal monopoly quantity for period 1, because retailers at least in expectation would then have no incentive to hold goods back for period 2, which would prevent inventory competition. The second term reflects the adjustment in the maximization condition that has to be made to account for the possibility of a stockout. This term implies that the static monopoly output is insufficient to maximize profit. Instead the manufacturer wants to choose a higher output in period 1, thus allowing some inventory competition in order to reduce the probability of a stockout. The probability of a stockout being less than 50% is then the by-product of shipping more than the static monopoly quantity in period 1.

The consequence of allowing some inventory competition in period 2 is not only that the manufacturer sets a relatively low  $P_2$ , but also that it ships a relatively small expected quantity in that period, one that is smaller than the static monopoly output. Overall, we obtain the following result for the manufacturer's expected total output and total profit:

**Proposition 2** *Suppose inventory is controlled by competitive retailers. The manufacturer's total expected equilibrium output,  $E(q^{cr}) \equiv q_1^{cr} + E(q_2^{cr}) = (A - c) + \frac{2d}{25}$ , and expected equilibrium profit,  $E(\Pi^{cr}) = \frac{(A-c)^2}{2} + \frac{d^2}{25}$ , exceed the sum over two periods of the static monopoly output, respectively static monopoly profit.*

**Proof:** See Appendix 2.

Notice that the equilibrium profit earned by the manufacturer is greater, namely by  $d^2/25$ , than the expected static monopoly profit over two periods.

This comes from the fact that, after observing  $\varepsilon_1$ , any excess inventory can be carried over to period 2, and the quantity ordered in period 2 can also be adjusted. But, as we showed in this section, there are two reasons why this profit is still lower than it could be. First, in order to lower the probability of stockout, the manufacturer ships a larger quantity in period 1. As will become evident below, this leads to an overall quantity that is too large to maximize the supply chain's value. Second, competitive retailers prevent optimal intertemporal price discrimination because they choose first-period sales to equalize the retail price in period 1 with the expected retail price in period 2. In the next section, we contrast this case with the one where the manufacturer uses an exclusive retailer to control inventory.

### 3 Inventory Control by an Exclusive Retailer

Consider now the case of a manufacturer selling to an exclusive retailer and using two-part tariffs. Thus the manufacturer sets a producer price,  $P_t$ , and a fixed payment (or transfer),  $T_t$ , in each period  $t = 1, 2$ .<sup>5</sup> This case is interesting for the following reasons. If the manufacturer used linear pricing, it would face qualitatively the same trade-off between inventory competition in period 2 and lost sales due to stockouts in period 1, simply because it would still have only one instrument at its disposal to deal with this trade-off. A two-part tariff, by providing an additional instrument to the manufacturer, gives it more scope to resolve the inventory competition problem without running the risk of stockouts. In addition, two-part tariffs imply that, in principle, there is no double marginalization when it deals with an exclusive retailer and that the entire expected profit generated by the supply chain goes to the manufacturer. It also implies that the manufacturer has nothing to gain from having more than one retailer. Thus, one retailer controlling inventory represents the opposite extreme with respect to dealing with competitive retailers. We want to show that distortions are still present.

The timing of moves is as before, except that the manufacturer now sets a two-part tariff each period. That is, at the beginning of period 1, the manufacturer sets a two-part tariff  $(P_1, T_1)$ , the exclusive retailer orders and takes possession of quantity  $q_1$ . Demand in period 1 is then revealed, and the retailer sells to consumers a quantity  $s_1 \leq q_1$ . In period 2, the manufacturer chooses the two-part tariff  $(P_2, T_2)$ , and the retailer orders a quantity  $q_2$ . Then demand in period 2 is revealed and the retailer sells  $s_2 \leq q_2 + (q_1 - s_1)$ .

The key difference with respect to the case of competitive retailers turns out to be that an exclusive retailer can guarantee itself a profit in period 2

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<sup>5</sup>In principle the manufacturer could also use two-part tariffs when it sells to competitive retailers. But perfect competition in retailing implies that the transfer in each case would be equal to zero in equilibrium.

of at least

$$\pi^{out} \equiv [A - (q_1 - s_1)](q_1 - s_1), \quad (6)$$

simply by selling unsold units carried over from period 1,  $(q_1 - s_1)$ , and refusing any additional orders in period 2. The exclusive retailer, it is important to notice, receives this profit, whether or not it orders output in period 2. As a result, if the manufacturer wants to sell output to the retailer in period 2, it has either to reduce the fixed fee it charges the retailer in period 2 by the amount  $\pi^{out}$ , or else to decrease its second-period producer price.

As it turns out, the manufacturer indeed finds it optimal to reduce the fixed fee by  $\pi^{out}$  and to set the producer price in period 2 equal to marginal cost  $c$ . This has the following implication:

**Proposition 3** *Suppose inventory is controlled by an exclusive retailer and the manufacturer uses a two-part tariff. If the retailer orders a positive quantity in period 2, then the expected retail price in period 2 is equal to the static monopoly price,  $E(p_2) = \frac{A+c}{2}$ . Hence inventory competition is eliminated.*

To prove this, suppose that the excess inventory from period 1 is small enough so that  $q_2 > 0$ . The retailer's expected profit in period 2 is then equal to  $(A - q_2 - (q_1 - s_1))(q_2 + (q_1 - s_1)) - P_2 q_2 - T_2$  so that its profit-maximizing order is

$$q_2 = \frac{A - P_2}{2} - (q_1 - s_1) > 0, \quad (7)$$

and the expected retail price is  $E(p_2) = \frac{A+P_2}{2}$ . To show that the manufacturer sets  $P_2 = c$ , it suffices to derive the retailer's expected profit in period 2,

$$E(\pi_2) = E(p_2)s_2 - P_2 q_2 - T_2 \quad (8)$$

$$= \frac{(A - P_2)^2}{4} + P_2(q_1 - s_1) - T_2, \quad (9)$$

and then to find the manufacturer's optimal two-part tariff  $(T_2, P_2)$  such that  $T_2$  extracts the retailer's expected period-2 profit net of the retailer's outside option,  $\pi^{out}$ , and  $P_2$  maximizes

$$\begin{aligned} & (P_2 - c)q_2(P_2) + E(\pi_2) - \pi^{out} \\ &= (P_2 - c) \left[ \frac{A - P_2}{2} - (q_1 - s_1) \right] + \frac{(A - P_2)^2}{4} + P_2(q_1 - s_1) - \pi^{out}. \end{aligned} \quad (10)$$

From the corresponding first-order condition, we obtain the manufacturer's optimal choice  $P_2 = c$ . Therefore the manufacturer's expected profit in period 2 conditional on  $q_2 > 0$  is

$$\begin{aligned} E(\Pi_2) &= \frac{(A - c)^2}{4} + c(q_1 - s_1) - \pi^{out} \\ &= \frac{(A - c)^2}{4} + [c - A + (q_1 - s_1)](q_1 - s_1). \end{aligned}$$

Thus, by setting the producer price in period 2 equal to the marginal cost  $c$ , the manufacturer induces the exclusive retailer to order the quantity that allows it to sell the expected static monopoly output in period 2 and to charge the corresponding static monopoly price. Thanks to the two-part tariff, the expected retail price in period 2 is independent of the excess inventory carried over from period 1. Hence, unlike in the case with competitive retailers, the manufacturer is able to eliminate inventory competition.

What does the retailer's guaranteed profit in period 2 imply for the choice of  $s_1$ , once the demand in period 1 has been revealed? Selling a unit in period 1 implies a realized marginal revenue of  $MR_1(\varepsilon_1) = A - 2s_1 + \varepsilon_1$ , whereas selling this unit in period 2 contributes to  $\pi^{out}$  and thus yields a marginal benefit equal to  $\frac{\partial \pi^{out}}{\partial (q_1 - s_1)} = A - 2(q_1 - s_1)$ . Hence the retailer chooses  $s_1$  so that

$$MR_1(\varepsilon_1) = \frac{\partial \pi^{out}}{\partial (q_1 - s_1)}. \quad (11)$$

We can solve (11) to obtain

$$s_1 = \frac{q_1}{2} + \frac{\varepsilon_1}{4}. \quad (12)$$

This shows that, because the retailer values excess inventory, it chooses to sell only half the quantity ordered in period 1 (plus  $\varepsilon_1/4$  which may be positive or negative).

The manufacturer selects the optimal two-part tariff  $(P_1, T_1)$  such that  $T_1$  extracts the retailer's total expected profit in period 1 and  $P_1$  maximizes

$$(P_1 - c)q_1(P_1) + T_1(P_1) + \int_{\hat{\varepsilon}_1(P_1)}^d \Pi_2 \frac{1}{2d} d\varepsilon_1. \quad (13)$$

Hence the manufacturer takes into account how its choice of  $P_1$  affects its expected profit in period 2 understanding that this expected profit depends on the retailer's re-ordering which occurs only if the demand shock in period 1 is bigger than the threshold  $\hat{\varepsilon}_1$ . Using the superscript 'ex' to denote equilibrium values in the case of an exclusive retailer, we prove:

**Proposition 4** *Suppose inventory is controlled by an exclusive retailer. In period 1, the manufacturer optimally sells  $q_1^{ex} = A - c - (\frac{3}{2} - \sqrt{2})d$ . This shipment is sufficiently large to avoid any possibility of stockout.*

**Proof:** See Appendix 3.

To understand this proposition, it is useful to rewrite the manufacturer's first-order condition coming from (13) (see also eq. (A.13) in Appendix 3) as:

$$-(P_1 - c) - \frac{1}{2} \int_{-(2P_1 - 2c)}^d \left( c - P_1 - \frac{\varepsilon_1}{2} \right) \frac{1}{2d} d\varepsilon_1 = 0. \quad (14)$$

When the retailer has no opportunity to re-order in period 2, the first-order condition reduces to the first term with the manufacturer setting  $P_1 = c$ . As a result,  $q_1 = A - c$  so that the retailer orders twice the static monopoly output in period 1. The second term reflects the effect of  $P_1$  on the manufacturer's expected profit when the retailer has an opportunity to re-order and thus when  $q_2 > 0$ . In Appendix 3, we formally show that  $q_2 > 0$  requires  $\varepsilon_1 > \hat{\varepsilon}_1 = -2(P_1 - c)$  and that it is this term that leads to  $P_1$  exceeding  $c$ . The intuition is clear. Raising  $P_1$  decreases  $T_1$  but increases the probability that the period-2 order is positive, thereby raising the manufacturer's expected profit in that period. We show that even with  $P_1$  exceeding  $c$  the retailer is still left with an incentive to place a large enough order in period 1 to avoid a stockout.

Now that we have determined how much the manufacturer ships in period 1, we can determine how much is shipped in equilibrium in period 2. Given sales in period 1 equal to  $s_1 = \frac{q_1}{2} + \frac{\varepsilon_1}{4}$  and desired expected sales in period 2 equal to  $s_2 = \frac{A-c}{2}$ , we have

$$\begin{aligned} q_2 &= s_2 - (q_1 - s_1) \\ &= \frac{A-c}{2} - (q_1 - s_1) \\ &= \frac{1}{4} \left[ (3 - 2\sqrt{2})d + \varepsilon_1 \right]. \end{aligned} \quad (15)$$

It follows that the retailer orders goods in period 2 ( $q_2 > 0$ ) only if demand in the first period is large enough such that  $\varepsilon_1 > \hat{\varepsilon}_1 = -(3 - 2\sqrt{2})d$ . If  $\varepsilon_1 < \hat{\varepsilon}_1$ , the retailer does not order any goods in period 2 and is content with the initial order. Accordingly, the expected second-period shipment by the manufacturer is

$$E(q_2^{ex}) = \int_{-(3-2\sqrt{2})d}^d \left[ \frac{(3 - 2\sqrt{2})d + \varepsilon_1}{4} \right] \frac{1}{2d} d\varepsilon_1 = \frac{(3 - 2\sqrt{2})}{2} d. \quad (16)$$

The manufacturer's total expected output and total expected profit can now be computed. They are as follows:

**Proposition 5** *Suppose inventory is controlled by an exclusive retailer. The manufacturer's equilibrium output,  $E(q^{ex}) \equiv q_1^{ex} + E(q_2^{ex}) = A - c$ , is the same as the sum over two periods of the static monopoly outputs, and the expected equilibrium profit,  $E(\Pi^{ex}) = \frac{(A-c)^2}{2} + \left(\frac{4\sqrt{2}-5}{12}\right) d^2$ , exceeds the sum of the static monopoly profits.*

**Proof:** See Appendix 4.

A comparison of the expected profit with that in the case of competitive retailers confirms that an exclusive retailer's incentives to manage inventory

are better aligned with the manufacturer's interest than those of competitive retailers. The reasons are that inventory competition is eliminated without incurring the risk of a stockout, and that the exclusive retailer is better at allocating inventory across periods, which improves intertemporal price discrimination.<sup>6</sup>

However, even if there is no longer any inventory competition and the exclusive retailer helps improve intertemporal price discrimination, there are still distortions. One is induced by the retailer's incentive to carry too much excess inventory into period 2 and thus to increase  $\pi^{out}$ . We can call this an *inventory rent distortion*. To see how it comes about, recall from (11) that the retailer chooses  $s_1$  to equate the marginal revenue from selling one more unit in period 1,  $MR_1(\varepsilon_1)$  with the marginal increase in the outside profit,  $\frac{\partial \pi^{out}}{\partial (q_1 - s_1)}$ . What the retailer should be doing to maximize the chain's total profit, as will be shown formally in the next section, is instead to choose  $s_1$  such that revenue on the marginal unit sold in period 1 is equal to the replacement cost of this unit, namely  $c$ . Hence a distortion arises as soon as  $\frac{\partial \pi^{out}}{\partial (q_1 - s_1)} \neq c$ . In particular, using (12), it is easy to see that a high demand shock ( $\varepsilon_1 > \hat{\varepsilon}_1$ ) implies  $\frac{\partial \pi^{out}}{\partial (q_1 - s_1)} > c$ ; this means that the retailer carries too much inventory into period 2, because it values a unit of excess inventory carried into period 2 at more than the producer price at which it could buy this unit in period 2, namely  $c$ . It is to reduce this distortion that the manufacturer raises  $P_1$  above  $c$ . While this increases the manufacturer's expected total profit, the resulting double marginalization constitutes a distortion of its own causing a deadweight loss.

## 4 Inventory Control by the Manufacturer

The previous section shows that distortions still exist when an exclusive retailer controls inventory because of the value that the retailer attaches to excess inventory carried into period 2. What then is the outcome if the manufacturer controls inventory itself, either in the sense that it is vertically integrated into retailing or in the sense that it lets competitive retailers set retail prices but controls the quantity shipped to consumers each period as would be the case, for instance, with the business practices discussed in the introduction, namely drop shipping, inventory consignment or vendor-managed inventory. Of course, production still needs to be committed before the demand is known, but  $q_t$  should no longer be interpreted as a retail order but rather as the manufacturer's production ready for sale before the

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<sup>6</sup>Notice, however, that inducing the retailer to allocate inventory better between periods requires shipping a large quantity in period 1 and letting the retailer carry large excess inventory into period 2. This solution is obviously facilitated by our assumption that there are no costs of holding inventory. Introducing such costs would reduce the advantage of having an exclusive retailer control inventory relative to competitive retailers.



demand is revealed in each period  $t = 1, 2$ .

Consider period 2. Given that  $d \leq \bar{d}$ , retail sales in period 2 equal  $s_2 = q_2 + (q_1 - s_1)$ , the expected profit of the manufacturer is  $(A - q_2 - (q_1 - s_1))(q_2 + (q_1 - s_1)) - cq_2$ , and the manufacturer's optimal production is

$$q_2 = \frac{A - c}{2} - (q_1 - s_1). \quad (17)$$

This implies that  $s_2 = \frac{A-c}{2}$  and the expected retail price is  $p_2 = \frac{A+c}{2}$  whether or not there is excess inventory at the end of period 1. The manufacturer's expected profit in period 2 is

$$E(\Pi_2) = \frac{(A - c)^2}{4} + c(q_1 - s_1), \quad (18)$$

where the unsold units carried over from period 1 are valued at their replacement cost  $c$ .

In period 1 after  $\varepsilon_1$  has been revealed, the manufacturer's marginal revenue is equal to  $MR_1(\varepsilon_1) = A - 2s_1 + \varepsilon_1$ , and it must decide how to allocate units across periods. We consider the case where  $q_2 > 0$ , which is satisfied for  $d \leq \bar{d}$ . The manufacturer's optimal choice of  $s_1$  is given by the intertemporal optimization condition

$$MR_1(\varepsilon_1) = c, \quad (19)$$

which states that the manufacturer values the marginal unit sold in period 1 at its replacement cost. Hence

$$s_1 = \frac{A - c}{2} + \frac{\varepsilon_1}{2}. \quad (20)$$

So what is the manufacturer's optimal choice of  $q_1$  before observing  $\varepsilon_1$ ? By producing in period 1 a quantity

$$q_1^m = \frac{A - c + d}{2}, \quad (21)$$

and in period 2, after observing  $\varepsilon_1$ , producing a quantity

$$\begin{aligned} q_2^m &= \frac{A - c}{2} - (q_1 - s_1) \\ &= \frac{A - c}{2} - \frac{d - \varepsilon_1}{2}, \end{aligned} \quad (22)$$

the manufacturer can make sure that it has enough inventory on hand to realize the optimal  $s_1$  and  $s_2$  for any realization of  $\varepsilon_1$ . This way the manufacturer can achieve a maximal total expected profit given by

$$E(\Pi^m) = \int_{-d}^d \frac{(A - c + \varepsilon_1)^2}{4} \frac{1}{2d} d\varepsilon_1 + \frac{(A - c)^2}{4}, \quad (23)$$

$$= \frac{(A - c)^2}{2} + \frac{d^2}{12}. \quad (24)$$

We may hence state:

**Proposition 6** *Suppose inventory is controlled by the manufacturer. The manufacturer's equilibrium output,  $E(q^m) = q_1^m + E(q_2^m) = A - c$ , is equal to the sum over two periods of the static monopoly outputs, and the expected equilibrium profit,  $E(\Pi^m) = \frac{(A-c)^2}{2} + \frac{d^2}{12}$ , exceeds the sum of the static monopoly profits.*

The advantage of having the manufacturer in control of inventory relative to an exclusive retailer comes directly from the intertemporal allocation condition (19). By equating marginal revenue in period 1 with the replacement cost (and thus implicitly with expected marginal revenue in period 2), the manufacturer can ensure that retail prices adjust optimally to difference in demand conditions across periods induced by different realizations of  $\varepsilon_1$ . That is, only in the case of manufacturer controlled inventory can price discrimination across periods be optimal.

## 5 Implications

We can now draw several implications from the analysis by comparing the equilibrium where competitive retailers control inventories with the equilibria where an exclusive retailer, respectively the manufacturer does. The first implication concerns the incentives to take inventory control away from competitive or exclusive retailers and assign it to the manufacturer. A comparison of Propositions 2, 5 and 6 shows, not surprisingly, that the manufacturer's profit is highest when it controls inventory itself, second highest when inventory control rests with an exclusive retailer and lowest when competitive retailers control inventory. More interestingly, the difference in profits between these scenarios becomes greater the greater is the variance of demand, and thus  $d$ . We hence obtain the following implication:

**Proposition 7** *The greater is the variance of demand (and thus  $d$ ) the greater is the incentive to let inventory be controlled by an exclusive retailer rather than competitive retailers and by the manufacturer rather than an exclusive retailer.*

Thus the distortions associated with having inventory controlled by competitive or exclusive retailers become greater the greater is the variance of demand. To see why, consider the impact of a big negative demand shock on inventory competition. In this case, competitive retailers are likely to carry a lot of excess inventory into the second period, even if the manufacturer only shipped a small quantity in period 1, thus exposing the manufacturer to inventory competition and forcing it to cut the producer price in period 2. In the case of a big positive demand shock the manufacturer faces another problem, namely that competitive retailers are likely to stock out and sales are lost. When inventory is controlled by an exclusive retailer, a comparison

of the intertemporal allocation rules (11) and (19) reveals that the exclusive retailer tends to carry too much excess inventory into period 2. The reason is simply that by doing so the retailer can increase inventory rent. Only when the manufacturer controls inventory can it ensure an optimal intertemporal allocation of inventory and thus achieve optimal intertemporal price discrimination; and this becomes more important the greater is the potential for demand differences across periods, i.e., the variance of demand shocks.

Another implication concerns the total expected output of the manufacturer and hence total inventory:

**Proposition 8** *Total inventory is the same when inventory is controlled by the manufacturer and by an exclusive retailer. Total inventory is highest when it is controlled by competitive retailers.*

Consider first the case of inventory control by competitive retailers. In this case, the manufacturer ships more than the static monopoly output in period 1 in order to reduce the probability of a stockout. The shipment in period 2 is smaller, but not small enough to compensate for the big shipment in period 1, simply because competitive retailers do not allocate inventory optimally between periods. An exclusive retailer helps eliminate inventory competition, but still cannot maximize the supply chain's value. This is because the exclusive retailer, too, does not allocate inventory optimally across periods. At the margin, the exclusive retailer has an incentive to sell too much in period 2 and too little in period 1, forcing the manufacturer to ship a large quantity in period 1. Interestingly, by raising  $P_1$  above the unit cost, the manufacturer forces the retailer to order less in period 1 and more in period 2 such that the manufacturer's expected total shipment is the same as when it fully controls inventory

The final implication is about consumer surplus and social welfare:

**Proposition 9** *Consumer surplus and social welfare are lower when inventory is controlled by an exclusive retailer rather than by the manufacturer, and they are highest when inventory is controlled by competitive retailers.*

**Proof:** See Appendix 5.

It is not surprising that reducing total expected output and thus inventory is bad for consumers and social welfare. The fact that, despite the same expected total output, consumer surplus and social welfare are lower when an exclusive retailer controls inventory than when the manufacturer does it reflects the deadweight loss that arises when the manufacturer raises the producer price in period 1 above marginal cost.

## 6 Conclusions

There is a general trend toward shifting inventory control away from retailers in many markets. This can be seen through the increasing popularity of business practices such as ‘drop-shipping’, ‘inventory consignment’ and ‘vendor-managed inventory’. These days, even Amazon is changing its business model from being a reseller to offering a platform for independent sellers and manufacturers with its Marketplace Sellers platform. Along with its ‘Fulfillment by Amazon’ it also gives the possibility to these sellers to let Amazon ship their products directly to consumers (Amazon, 2014). This trend can also be seen more indirectly through the growth of logistics firms. Since the early 2000s, this market has seen the emergence and the rapid growth of the ‘value-added warehouse distribution providers’ (for instance, Excel, UPS SCS, Kenco, Genco, Jacobson and DSC; see Foster and Armstrong, 2004). These providers who combine logistics services with full service solutions including inventory management have now been largely absorbed by large 3PL providers.<sup>7</sup> Like for Amazon’s Marketplace Sellers, the success of these providers would not have occurred without a shift of inventory control away from retailers.

While this evidence may reflect a goal of having ‘just-in-time’ delivery of goods, it is important to ascertain the impact of shifting inventory control away from retailers on the functioning and on the efficiency of markets. This paper shows that shifting inventory control away from retailers may be an optimal strategy to follow for manufacturers in an environment in which orders must be placed before demand is known. This would be the case, even if this was achieved through the addition of a costly logistics provider. This is because shifting inventory control to the manufacturer or a designated logistics firm brings two advantages to inventory control, both of which stem from better incentives to allocate inventory over time. First, it essentially facilitates price discrimination across periods by intertemporally segmenting markets. Second, it helps eliminate inventory competition that would occur if inventory were controlled by competitive retailers. Both advantages are shown to be especially important in markets where demand is very volatile.

A number of implications follow from the analysis. An important one is that shifting inventory control away from retailers tends to be anti-competitive, at least when these retailers are competitive. In the context of our model with two sales periods and without destructive competition, competitive retailers have a tendency to order too much, simply because they know that their order, even if it may not be entirely sold in the first sales period, still has value in the second one. This is in sharp contrast to a one-period envi-

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<sup>7</sup>Excel has become DHL Supply Chain in 2016; Genco has been absorbed by FedEx in 2015; Jacobson was absorbed by Norbert Dentressangle in 2014 which in turn was absorbed by XPO Logistics in 2015; Kenco, a North American provider, has partnered with Hermes, a large European logistics provider for its international business.

ronment with destructive competition as in Deneckere et al. (1997). In that environment, competitive retailers are left only with the option of dumping the products at the end of the sales period in case of low demand. Not surprisingly this leads the retailers to order too little unless the manufacturer provides them with some guarantee, such as with a buy-back policy, that it will compensate retailers for excess inventory. By contrast, our manufacturer would like the retailers to buy less, not more, and relieving them of inventory control is a way to achieve this.

Since the literature on the theory of contracts in supply chains shows that vertical restraints and other policies can help to achieve, at least in principle, maximal vertical value, one could conclude that, instead of shifting inventory control away from retailers, the same outcome could also be achieved by imposing some vertical restraint, as long as it restricts the volume of orders placed by retailers. Although the precise characterization of such a vertical restraint is not part of the analysis, it still suggests that the choice between these two options very much depends on the specific conditions under which supply chains operate. In that regard the rapid growth of the business practices noted above demonstrates that shifting inventory control away from retailers might be an especially useful way to solve incentive problems in many markets.

Although it is beyond the scope of the present article to test our results empirically, testing them is potentially feasible. For instance, it is interesting to note that some of our theoretical predictions are consistent with the empirical results about drop-shipping provided by Randall, Netessine and Rudi (2006). The authors compare drop-shipping to the more traditional arrangement where retailers hold their own inventories. This is a similar structure to ours in so far as the drop-shipping arrangement corresponds to the case where a manufacturer (or a wholesaler) takes over inventory control from retailers. The authors find empirical evidence that traditional retailers who manage their own inventories face lower demand uncertainty than the retailers that rely on drop-shippers to control inventory. This is consistent with our result that shifting inventory control away from retailers is especially beneficial when there is high demand uncertainty. They also find that the greater the number of retailers, the greater is the use of drop-shipping. Although our retailers are either exclusive or perfectly competitive and thus we have no particular result on that dimension, it is interesting to note that the fundamental reason why a manufacturer might do better than a large number of retailers is that these retailers, as price takers, do not have the right incentives to allocate inventory over time. In that sense this empirical finding is also consistent with our theoretical results.

## Appendix

### 1 Proof of Proposition 1

Being perfectly competitive, retailers order goods in period 1 until the expected retail price is equal to their marginal cost, which in this case is the producer price  $P_1$ :

$$\int_{-d}^{\hat{\varepsilon}_1} \frac{A + c - q_1 + s_1}{2} \frac{1}{2d} d\varepsilon_1 + \int_{\hat{\varepsilon}_1}^d (A - q_1 + \varepsilon_1) \frac{1}{2d} d\varepsilon_1 = P_1. \quad (\text{A.1})$$

The first term in (A.1) is the expected retail price in period 1 if there is no stockout that we know from (2); the second term is the expected retail price in case of a stockout. Substituting for  $s_1$ , we can rewrite (A.1) as

$$\int_{-d}^{\hat{\varepsilon}_1} \frac{2A + c + \varepsilon_1 - q_1}{3} \frac{1}{2d} d\varepsilon_1 + \int_{\hat{\varepsilon}_1}^d (A + \varepsilon_1 - q_1) \frac{1}{2d} d\varepsilon_1 = P_1. \quad (\text{A.2})$$

The manufacturer chooses  $q_1$  to maximize total expected profit over the two periods, which is given by

$$\begin{aligned} (P_1 - c)q_1 + (P_2(q_1) - c)q_2(q_1) = \\ \int_{-d}^{\hat{\varepsilon}_1} \left[ \frac{2(A - c) + \varepsilon_1 - q_1}{3} q_1 + \frac{(2(A - c) + \varepsilon_1 - q_1)^2}{9} \right] \frac{1}{2d} d\varepsilon_1 \\ + \int_{\hat{\varepsilon}_1}^d \left[ (A - c + \varepsilon_1 - q_1) q_1 + \frac{(A - c)^2}{4} \right] \frac{1}{2d} d\varepsilon_1. \end{aligned} \quad (\text{A.3})$$

Using the Leibniz Rule, we can write the first-order condition associated with (A.3) as

$$\int_{-d}^{\hat{\varepsilon}_1} \frac{2(A - c) + \varepsilon_1 - 4q_1}{9} \frac{1}{2d} d\varepsilon_1 + \int_{\hat{\varepsilon}_1}^d (A - c + \varepsilon_1 - 2q_1) \frac{1}{2d} d\varepsilon_1 + X = 0 \quad (\text{A.4})$$

where

$$\begin{aligned} X = & \left[ \frac{2(A - c) + \hat{\varepsilon}_1 - q_1}{3} q_1 + \frac{(2(A - c) + \hat{\varepsilon}_1 - q_1)^2}{9} \right] \frac{d\hat{\varepsilon}_1}{dq_1} \\ & - \left[ (A - c + \hat{\varepsilon}_1 - q_1) q_1 + \frac{(A - c)^2}{4} \right] \frac{d\hat{\varepsilon}_1}{dq_1}. \end{aligned}$$

Noting that  $\frac{d\hat{\varepsilon}_1}{dq_1} = 1$  and substituting for  $\hat{\varepsilon}_1$  from (4) it is easily shown that  $X = 0$ , so that we can rewrite (A.4) as

$$\int_{-d}^d \frac{2(A-c) + \varepsilon_1 - 4q_1}{9} \frac{1}{2d} d\varepsilon_1 + \int_{\hat{\varepsilon}_1}^d \left[ (A-c + \varepsilon_1 - 2q_1) - \frac{2(A-c) + \varepsilon_1 - 4q_1}{9} \right] \frac{1}{2d} d\varepsilon_1 = 0, \quad (\text{A.5})$$

which, after simplification, becomes the first-order condition (5) in the main body of the paper.

After substitution for  $\hat{\varepsilon}_1$  and integration we can rewrite (A.5) to obtain

$$(A-c + 4d - 2q_1)(5(A-c) + 2d - 10q_1) = 0.$$

There are two solutions:  $q_1 = \frac{1}{2}(A-c) + \frac{1}{5}d$  and  $q_1 = \frac{1}{2}(A-c) + 2d$ . Since we require  $d > \hat{\varepsilon}_1 = q_1 - \frac{1}{2}(A-c)$ , only the first solution is valid. Hence

$$q_1^{cr} = \frac{1}{2}(A-c) + \frac{1}{5}d.$$

Using this level of output we obtain  $\hat{\varepsilon}_1 = \frac{1}{5}d$ . The probability of a stockout can then be computed as  $\frac{d - \frac{1}{5}d}{2d} = 0.4$ .

## 2 Proof of Proposition 2

The expected second-period output can be computed using  $q_2 = \frac{A-c-(q_1-s_1)}{2}$  if  $q_1 > s_1$ , and  $q_2 = \frac{A-c}{2}$  if there is a stockout. This implies:

$$\begin{aligned} E(q_2^{cr}) &= \int_{-d}^{\hat{\varepsilon}_1} \left[ \frac{2(A-c) - q_1 + \varepsilon_1}{3} \right] \frac{1}{2d} d\varepsilon_1 + \int_{\hat{\varepsilon}_1}^d \left( \frac{A-c}{2} \right) \frac{1}{2d} d\varepsilon_1 \quad (\text{A.6}) \\ &= \frac{1}{2}(A-c) - \frac{3}{25}d. \end{aligned} \quad (\text{A.7})$$

Total expected output is thus given by  $q_1^{cr} + E(q_2^{cr}) = \frac{1}{2}(A-c) + \frac{2}{25}d$ .

The manufacturer's total expected profit can be computed from (A.3)

as

$$\begin{aligned}
& \int_{-d}^{\hat{\varepsilon}_1} \left[ \frac{2(A-c) + \varepsilon_1 - \left(\frac{1}{2}(A-c) + \frac{1}{5}d\right)}{3} \left(\frac{1}{2}(A-c) + \frac{1}{5}d\right) \right] \frac{1}{2d} d\varepsilon_1 \\
& \quad + \int_{-d}^{\hat{\varepsilon}_1} \left[ \frac{\left(2(A-c) + \varepsilon_1 - \left(\frac{1}{2}(A-c) + \frac{1}{5}d\right)\right)^2}{9} \right] \frac{1}{2d} d\varepsilon_1 \\
& + \int_{\hat{\varepsilon}_1}^d \left[ \left(A - c + \varepsilon_1 - \left(\frac{1}{2}(A-c) + \frac{1}{5}d\right)\right) \left(\frac{1}{2}(A-c) + \frac{1}{5}d\right) + \frac{(A-c)^2}{4} \right] \frac{1}{2d} d\varepsilon_1,
\end{aligned}$$

which simplifies to

$$E(\Pi^{cr}) = \frac{(A-c)^2}{2} + \frac{d^2}{25}. \quad (\text{A.8})$$

### 3 Proof of Proposition 4

Since the sales and orders in period 2 have already been derived in connection with Proposition 3, we concentrate on period 1. In period 1, after  $\varepsilon_1$  has been revealed, the retailer's optimal choice of  $s_1$  is given by (12). Hence we have  $q_2 = \frac{A-c}{2} - (q_1 - s_1) = \frac{2(A-c) - 2q_1 + \varepsilon_1}{4}$ .

We obtain  $q_2 = 0$  if  $\varepsilon_1 < \hat{\varepsilon}_1 = 2q_1 - 2(A-c)$ . In this case, the retailer allocates inventory  $q_1$  to cover both periods, i.e.  $s_1 + s_2 = q_1$ , and the optimal choice of  $s_1$  satisfies  $MR_1(\varepsilon_1) = E[MR_2(\varepsilon_2)] = A - 2(q_1 - s_1)$ . This implies that the retailer's optimal choice of  $s_1$  is again given by (12), and  $s_2 = (2q_1 - \varepsilon_1)/4$ .

The retailer's total expected profit is then given by

$$E\{(A - s_1 + \varepsilon_1)s_1 + \pi^{out} - P_1q_1 - T_1\}, \quad (\text{A.9})$$

which can be rewritten as

$$\int_{-d}^d \left[ (A - q_1)q_1 + \frac{1}{8}(2q_1 + \varepsilon_1)^2 \right] \frac{1}{2d} d\varepsilon_1 - P_1q_1 - T_1. \quad (\text{A.10})$$

From the first-order condition,

$$\int_{-d}^d \left( A - q_1 + \frac{\varepsilon_1}{2} \right) \frac{1}{2d} d\varepsilon_1 - P_1 = 0, \quad (\text{A.11})$$

we obtain the optimal order quantity  $q_1 = A - P_1$  and the total expected profit of the retailer is

$$\frac{(A - P_1)^2}{2} + \frac{d^2}{24} - T_1. \quad (\text{A.12})$$



Since the manufacturer captures the retailer's total expected profit through  $T_1$ , the manufacturer chooses  $P_1$  to maximize

$$\begin{aligned} & \max_{P_1} (P_1 - c)(A - P_1) + \frac{(A - P_1)^2}{2} + \frac{d^2}{24} \\ & + \int_{\hat{\varepsilon}_1}^d \left[ \frac{(A - c)^2}{4} + c(q_1 - s_1) - \pi^{out} \right] \frac{1}{2d} d\varepsilon_1, \end{aligned}$$

where the first line corresponds to the manufacturer's expected total profit in period 1, and the second line is the manufacturer's expected total profit in period 2 (thus when  $q_2 > 0$ ) taking into account that  $P_2 = c$ .

Notice that in the second line a change in  $P_1$  impacts  $\hat{\varepsilon}_1$ ,  $(q_1 - s_1)$  and  $\pi^{out}$ , but  $\frac{(A-c)^2}{4} + c(q_1 - s_1) - \pi^{out} = 0$  for  $\varepsilon_1 = \hat{\varepsilon}_1$ . Thus, using the Leibniz rule and after simplifications, the first-order condition can be written as

$$-(P_1 - c) - \frac{1}{2} \int_{-2(P_1 - c)}^d \left( c - P_1 - \frac{\varepsilon_1}{2} \right) \frac{1}{2d} d\varepsilon_1 = 0, \quad (\text{A.13})$$

where  $\hat{\varepsilon}_1$  has been replaced by  $-2(P_1 - c)$ . This is because at  $\hat{\varepsilon}_1$  we have  $q_2 = 0$  and thus  $s_2 = q_1 - s_1$ ; hence with  $s_2 = \frac{A-c}{2}$  and  $q_1 - s_1 = \frac{q_1}{2} - \frac{\hat{\varepsilon}_1}{4} = \frac{A-P_1}{2} - \frac{\hat{\varepsilon}_1}{4}$ , it follows that  $\hat{\varepsilon}_1 = -2(P_1 - c)$ .

Solving for  $P_1$ , we obtain  $P_1 = c + \left( \frac{3-2\sqrt{2}}{2} \right) d$ . The manufacturer's output in period 1 is  $q_1^{ex} = A - c - \left( \frac{3-2\sqrt{2}}{2} \right) d$ , and the threshold value above which  $q_2$  is positive is  $\hat{\varepsilon}_1 = -\left( 3 - 2\sqrt{2} \right) d$ . Finally, since  $s_1 = \frac{q_1}{2} + \frac{\varepsilon_1}{4}$ , then  $q_1^{ex} > s_1(\varepsilon_1 = d)$  whenever  $A - c > d(2 - \sqrt{2})$ . This inequality holds for  $d < \bar{d}$ . Thus the exclusive retailer faces no stockout.

#### 4 Proof of Proposition 5

Adding  $q_1^{ex}$  and  $E(q_2^{ex})$  (from (15)), it is immediate that  $E(q^{ex}) = A - c$ .

The manufacturer's expected total profit when dealing with an exclusive retailer is given by (13) which can be rewritten as

$$\begin{aligned} & (P_1 - c)(A - P_1) + \frac{(A - P_1)^2}{2} + \frac{d^2}{24} \\ & + \int_{\hat{\varepsilon}_1}^d \left[ \frac{(A - c)^2}{4} + c(q_1 - s_1) - \pi^{out} \right] \frac{1}{2d} d\varepsilon_1. \end{aligned}$$

Substituting for  $\pi^{out}$  from (6), using  $P_1 = c + \left( \frac{3-2\sqrt{2}}{2} \right) d$ ,  $s_1 = \frac{q_1^{ex}}{2} + \frac{\varepsilon_1}{4}$ ,  $\hat{\varepsilon}_1 = -\left( 3 - 2\sqrt{2} \right) d$  and  $q_1^{ex} = A - c - \left( \frac{3}{2} - \sqrt{2} \right) d$ , it can, after some manipulations,

be rewritten as

$$\begin{aligned} E(\Pi^{ex}) &= \frac{1}{2}(A-c)^2 - \left(\frac{17-12\sqrt{2}}{8}\right)d^2 + \frac{d^2}{24} + \frac{10-7\sqrt{2}}{6}d^2 \\ &= \frac{1}{2}(A-c)^2 + \left(\frac{4\sqrt{2}-5}{12}\right)d^2. \end{aligned}$$

## 5 Proof of Proposition 9

When inventory is controlled by competitive retailers, consumer surplus is given by:

$$CS^{cr} = \frac{1}{2}E[(s_1^{cr})^2] + \frac{1}{2}E[(s_2^{cr})^2],$$

where

$$\begin{aligned} &\frac{1}{2}E[(s_1^{cr})^2] \\ &= \frac{1}{2} \int_{-d}^{\frac{1}{5}d} \left(\frac{1}{2}(A-c) + \frac{2}{3}\varepsilon_1 + \frac{1}{15}d\right)^2 \frac{1}{2d}d\varepsilon_1 + \frac{1}{2} \int_{\frac{1}{5}d}^d \left(\frac{1}{2}(A-c) + \frac{1}{5}d\right)^2 \frac{1}{2d}d\varepsilon_1 \\ &= \frac{1}{8}(A-c)^2 - \frac{1}{50}d(A-c) + \frac{9}{250}d^2, \end{aligned}$$

and

$$\begin{aligned} &\frac{1}{2}E[(s_2^{cr})^2] \\ &= \frac{1}{2} \int_{-d}^{\frac{1}{5}d} \left(\frac{A-c}{2} + \frac{d}{15} - \frac{\varepsilon_1}{3}\right)^2 \frac{1}{2d}d\varepsilon_1 + \frac{1}{2} \int_{\frac{1}{5}d}^d \left(\frac{A-c}{2}\right)^2 \frac{1}{2d}d\varepsilon_1 \\ &= \frac{1}{8}(A-c)^2 + \frac{3}{50}d(A-c) + \frac{2}{125}d^2. \end{aligned}$$

Hence the consumer surplus is

$$CS^{cr} = \frac{1}{4}(A-c)^2 + \frac{1}{25}(A-c)d + \frac{13}{250}d^2. \quad (\text{A.14})$$

When inventory is controlled by an exclusive retailer, the consumer surplus is equal to

$$CS^{ex} = \frac{1}{2}E[(s_1^{ex})^2] + \frac{1}{2}E[(s_2^{ex})^2],$$

where

$$\begin{aligned}\frac{1}{2}E[(s_1^{ex})^2] &= \frac{1}{2}\int_{-d}^d \left( \frac{A-c - \left(\frac{3}{2} - \sqrt{2}\right)d}{2} + \frac{\varepsilon_1}{4} \right)^2 \frac{1}{2d} d\varepsilon_1 \\ &= \frac{1}{8} \left( A-c - \left( \frac{3}{2} - \sqrt{2} \right) d \right)^2 + \frac{1}{96} d^2,\end{aligned}$$

and

$$\begin{aligned}\frac{1}{2}E[(s_2^{ex})^2] &= \frac{1}{2}\int_{-d}^{\hat{\varepsilon}_1} \left( \frac{A-c - \left(\frac{3}{2} - \sqrt{2}\right)d}{2} - \frac{\varepsilon_1}{4} \right)^2 \frac{1}{2d} d\varepsilon_1 \\ &\quad + \frac{1}{2}\int_{\hat{\varepsilon}_1}^d \left( \frac{A-c}{2} \right)^2 \frac{1}{2d} d\varepsilon_1 \\ &= \frac{1}{2}\int_{-d}^d \left( \frac{A-c}{2} \right)^2 \frac{1}{2d} d\varepsilon_1 \\ &\quad + \frac{1}{2}\int_{-d}^{\hat{\varepsilon}_1} \left[ -\frac{1}{2}(A-c) \left( \frac{3}{2} - \sqrt{2} \right) d + \frac{1}{4} \left( \frac{3}{2} - \sqrt{2} \right)^2 d^2 \right] \frac{1}{2d} d\varepsilon_1 \\ &\quad + \frac{1}{2}\int_{-d}^{\hat{\varepsilon}_1} \left[ -\frac{[A-c - \left(\frac{3}{2} - \sqrt{2}\right)d]\varepsilon_1}{4} + \frac{(\varepsilon_1)^2}{16} \right] \frac{1}{2d} d\varepsilon_1 \\ &= \frac{1}{8}(A-c)^2 + \frac{(7-5\sqrt{2})(A-c)d}{8} + \frac{(29\sqrt{2}-41)d^2}{32} \\ &\quad + \frac{35\sqrt{2}-49}{96}d^2 - \frac{[A-c - \left(\frac{3}{2} - \sqrt{2}\right)d](16-12\sqrt{2})d}{32}.\end{aligned}$$

Thus

$$\begin{aligned}CS^{ex} &= \frac{1}{2}E[(s_1^{ex})^2] + \frac{1}{2}E[(s_2^{ex})^2] \\ &= \frac{1}{4}(A-c)^2 + \frac{3-2\sqrt{2}}{12}d^2.\end{aligned}\tag{A.15}$$

The consumer surplus when inventory is controlled by the manufacturer is equal to:

$$\begin{aligned}CS^m &= \frac{1}{2}\int_{-d}^d \left( \frac{A-c + \varepsilon_1}{2} \right)^2 \frac{1}{2d} d\varepsilon_1 + \frac{1}{2}\int_{-d}^d \left( \frac{A-c}{2} \right)^2 \frac{1}{2d} d\varepsilon_1 \\ &= \frac{(A-c)^2}{4} + \frac{d^2}{24}.\end{aligned}\tag{A.16}$$

Comparing consumer surpluses:

$$\begin{aligned} CS^{cr} - CS^m &= \frac{(A-c)}{4}d + \frac{31}{3000}d^2 > 0 \\ CS^m - CS^{ex} &= \frac{(5\sqrt{2}-6)}{24}d^2 > 0. \end{aligned}$$

Thus,

$$CS^{cr} > CS^m > CS^{ex} \text{ for any } d < \bar{d}.$$

Next we compute social welfare in the three scenarios:

$$SW^{cr} = CS^{cr} + E(\Pi^{cr}) = \frac{3}{4}(A-c)^2 + \frac{1}{25}d(A-c) + \frac{23}{250}d^2 \quad (\text{A.17})$$

$$SW^{ex} = CS^{ex} + E(\Pi^{ex}) = \frac{3}{4}(A-c)^2 + \frac{\sqrt{2}-1}{6}d^2 \quad (\text{A.18})$$

$$SW^m = CS^m + E(\Pi^m) = \frac{3}{4}(A-c)^2 + \frac{3}{24}d^2. \quad (\text{A.19})$$

A comparison of social welfare across the scenarios yields:

$$SW^{cr} > SW^m > SW^{ex} \text{ for any } d < \bar{d}.$$

## 6 Derivation of $\bar{d}$

Inventory Control by Competitive Retailers: In equilibrium,  $q_1 = \frac{1}{2}(A-c) + \frac{1}{5}d$ . Thus  $s_1 = \frac{A-c+2\varepsilon_1}{3} + \frac{1}{3}q_1 = \frac{1}{2}(A-c) + \frac{2}{3}\varepsilon_1 + \frac{1}{15}d$  and  $s_2 = \frac{A-c+(q_1-s_1)}{2}$ . We know that if  $\varepsilon_1 < \frac{1}{5}d$ , then  $q_1 > s_1$ ; otherwise there will be a stockout. Obviously  $s_2$  is always larger than zero. To ensure  $s_1 > 0$ , we require  $d < \frac{5}{6}(A-c)$ . In both cases,  $q_2 = \frac{A-c-q_1+s_1}{2}$ ; hence  $\forall \varepsilon_1$  we have  $q_2 > 0$  if  $d < \frac{5(A-c)}{4}$ .

The retail price in period 1 is given by  $p_1 = A - s_1 + \varepsilon_1 = \frac{1}{2}(A+c) + \frac{1}{3}\varepsilon_1 - \frac{1}{15}d$ . Hence  $\forall \varepsilon_1$  we have  $p_1 > 0$  if  $d < \frac{5(A-c)}{4}$ . The retail price in period 2 is  $p_2 = A - s_2 + \varepsilon_2 = \frac{A+c}{2} - \frac{1}{15}d + \frac{1}{3}\varepsilon_1 + \varepsilon_2$ . Thus  $\forall \varepsilon_1, \varepsilon_2$  we obtain  $p_2 > 0$ , if  $d < \frac{5(A+c)}{14}$ . Notice that  $p_2 > 0$  implies that there is no destructive competition and that all inventory is sold at in period 2.

Inventory Control by an Exclusive Retailer: In equilibrium,  $q_1 = A - c - \left(\frac{3}{2} - \sqrt{2}\right)d$ ,  $s_1 = \frac{A-c-\left(\frac{3}{2}-\sqrt{2}\right)d}{2} + \frac{\varepsilon_1}{4}$ . If  $\varepsilon_1 > -\left(3-2\sqrt{2}\right)d$ , then  $q_2 = \frac{(3-2\sqrt{2})d+\varepsilon_1}{4} > 0$ ; otherwise  $q_2 = 0$  and  $s_2 = q_2 + (q_1 - s_1) = q_2 + \frac{A-c-\left(\frac{3}{2}-\sqrt{2}\right)d}{2} - \frac{\varepsilon_1}{4}$  is always greater than zero. To make sure  $s_1 > 0$ , we require  $d < \frac{(A-c)}{2-\sqrt{2}}$ . There is no stockout in period 1 if  $q_1 > s_1 \forall \varepsilon_1$ , which is satisfied for  $d < \frac{(A-c)}{2-\sqrt{2}}$ .

If  $q_2 > 0$ ,  $s_2 = \frac{A-c}{2}$ , and  $MR_2 > 0$  requires  $d < c$ . If  $q_2 = 0$ , then  $s_2 = \frac{A-c - (\frac{3}{2} - \sqrt{2})d}{2} - \frac{\varepsilon_1}{4}$ , and  $MR_2 > 0$  requires  $d < \frac{c}{\sqrt{2}}$ . One can check that if  $d < \frac{c}{\sqrt{2}-1}$ , then  $MR_1 > 0$  and  $p_1, p_2 > 0$  given that  $s_1, s_2 > 0$ .

Inventory Control by the Manufacturer: In equilibrium,  $q_1 = \frac{A-c+d}{2}$ ,  $s_1 = \frac{A-c+\varepsilon_1}{2}$ ; and  $q_2 = \frac{A-c}{2} - \frac{d-\varepsilon_1}{2}$ ,  $s_2 = \frac{A-c}{2}$ . Then  $q_1, s_1, q_2 > 0$  as long as  $d < \frac{A-c}{2}$ . In addition,  $MR_1 = c$  and  $MR_2 = c + \varepsilon_2 > 0$  requires  $d < c$ .

Sufficient Conditions: All sufficient conditions are satisfied if

$$d < \min \left[ \frac{5(A-c)}{4}, \frac{5(A+c)}{14}, \frac{c}{\sqrt{2}}, \frac{A-c}{2} \right].$$

Since  $A > c$ , this can be simplified to  $d < \bar{d} = \min \left[ \frac{c}{\sqrt{2}}, \frac{A-c}{2} \right]$ .

## References

- [1] Amazon, 2014. Amazon Enjoys Record-Setting Year for Marketplace Sellers, <http://www.newswire.ca/news-releases/amazon-enjoys-record-setting-year-for-marketplace-sellers-513532481.html>
- [2] Arrow, Kenneth, Theodore Harris and Jacob Marschak, 1951. Optimal Inventory Policy, *Econometrica* 25, 522-52.
- [3] Bulow, Jeremy, 1982. Durable Goods Monopolist, *Journal of Political Economy* 90, 314-332.
- [4] Cachon, Gérard P., 2004. The allocation of inventory risk in a supply chain: Push, pull, and advance-purchase discount contracts, *Management Science* 50(2), 222-238.
- [5] Clark, Andrew and Herbert Scarf, 1960. Optimal Policies for a Multi-Echelon Inventory Problem, *Management Science* 6, 475-90.
- [6] Deneckere, Raymond, Howard P. Marvel, and James Peck, 1996. Demand Uncertainty, Inventories, and Resale Price Maintenance. *Quarterly Journal of Economics* 111(3), 885-913.
- [7] Deneckere, Raymond, Howard P. Marvel, and James Peck, 1997. Demand Uncertainty and Price Maintenance: Markdowns as Destructive Competition, *American Economic Review* 87(4), 619-41.
- [8] Dudine, Paolo, Igal Hendel and Alessandro Lizzeri, 2006. Storable Good Monopoly: The Role of Commitment. *American Economic Review* 96(5), 1706-1719.

- [9] Foster, Thomas and Richard Armstrong, 2004. Top 25 Third Party Logistics Providers Extend Their Global Reach, *Supply Chain Brain*, <http://www.supplychainbrain.com/content/sponsored-channels/kenco-logistic-services-third-party-logistics/single-article-page/article/top-25-third-party-logistics-providers-extend-their-global-reach/>
- [10] Govindam, Kannan, 2013, Vendor-Managed Inventory: A Review based on Dimensions, *International Journal of Production Research* 51(13), 3808-35.
- [11] Koh, W. T., 2006. The micro-foundations of intertemporal price discrimination, *Economic Theory*, 27(2), 393-410.
- [12] Krishnan, Harish and Ralph Winter, 2011. The Economic Foundations of Supply Chain Contracting, *Foundations and Trends in Technology, Information and Operations Management* 5, 3-4, 147-309.
- [13] Krishnan, Harish and Ralph Winter, 2010. Inventory Dynamics and Supply Chain Coordination, *Management Science* 56(1), 141-147.
- [14] Krishnan, Harish and Ralph Winter, 2007. Vertical Control of Price and Inventory, *American Economic Review* 97(5), 1840-57.
- [15] Mateen, Arqum and Ashis K. Chatterjee, 2015. Vendor-Managed Inventory for Single-Vendor Multi-Retailers Supply Chains, *Decisions Support Systems* 70, 31-41.
- [16] Randall, Taylor, Netessine, Sergei and Nils Rudi, 2006, An Empirical Examination of the Decision to Invest in Fulfillment Capabilities: A Study of Internet Retailers, *Management Science* 52(4), 567-80.
- [17] Randall, Taylor, Netessine, Sergei and Nils Rudi, 2002. Should you take the virtual fulfillment path? *Supply Chain Management Review*, November/December, 54-58.
- [18] Tirole, Jean, 1988. *The Theory of Industrial Organization*, MIT Press, Cambridge (Mass.).
- [19] Varian, Hal R., 1989. Price discrimination, *Handbook of industrial organization* 1: 597-654.